also apply for diffusion processes, and one can, in similar fashion, define a Fick's type medium with an infinite mass propagation velocity and a Maxwellian medium with a finite mass propagation velocity. The results obtained here are valid for mass-transfer processes in these materials.

NOTATION

λ_0, equilibrium thermal conductivity; ρ_0, mass density, c_0, equilibrium heat capacity of the material; $\lambda(t)$, $c(t)$, relaxation kernels for the heat flux and internal energy; $a_0 = \lambda_0/\rho_0 c_0$, thermal diffusivity, T, temperature; L, L^{-1}, Laplace transform and inverse Laplace transform operators; p, Laplace transform variable; $\lambda_1(t) = \lambda(t)/\lambda_0$, $c_1(t) = c(t)/c_0\rho_0$, dimensionless relaxation kernels for the heat flux and internal energy; M, spatial point; J_1, I_1, first-order Bessel functions of the first kind for real and imaginary argument; J_0, I_0, zeroth-order Bessel functions of the first kind for real and imaginary argument; w, heat propagation velocity; i, imaginary unit; Re, real part of a complex number or function; σ, index of increase of the function u_1; $\delta(t)$, the Dirac delta function; E(t), Heaviside unit step function.

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TEMPERATURE DISTRIBUTION IN PLATES AND INFINITE PRISMATIC BODIES OF COMPLEX CROSS SECTION FOR A TIME-VARYING HEAT-TRANSFER COEFFICIENT

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We present a new method for solving heat-conduction problems with a time-varying heat-transfer coefficient in domains of complex shape, and cite numerical results for two problems.

Because of mathematical difficulties, heat-conduction problems with a time-dependent heat-transfer coefficient cannot be solved analytically in complex domains for a given Bi(Fo), even for one-dimensional cases [1].

We consider the case when the calculation of the temperature distribution in plates and infinite prismatic bodies of complex cross section reduces to the solution of the heat-conduction problem

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TABLE 1. Values of the Temperature Calculated by Various Methods

Fo	T (0)			T (1)		
	1	2	3	1	2	3
0,1	0,1574	—	—	0,2954	—	—
0,2	0,1816	—	—	0,3587	—	—
0,3	0,2156	—	—	0,4104	—	—
0,4	0,2538	0,2748	0,2854	0,4592	0,4577	0,4823
0,5	0,2944	0,3256	0,3326	0,5072	0,5228	0,5358
0,6	0,3366	0,3599	0,3666	0,5553	0,5671	0,5781
0,7	0,3800	0,4063	0,4122	0,6031	0,6179	0,6275
0,8	0,4245	0,4439	0,4482	0,6504	0,6610	0,6693
0,9	0,4695	0,4948	0,4987	0,6968	0,7088	0,7158
1,0	0,5150	0,5238	0,5266	0,7420	0,7510	0,7551
1,1	0,5604	0,6061	0,6075	0,7855	0,8082	0,8123
1,2	0,6056	0,6461	0,6461	0,8272	0,8462	0,8487
1,3	0,6502	0,6933	0,6933	0,8666	0,8864	0,8864
1,4	0,6938	0,7422	0,7422	0,9037	0,9224	0,9224
1,5	0,7362	0,7701	0,7701	0,9383	0,9556	0,9556
1,6	0,7771	0,7947	0,7947	0,9702	0,9803	0,9803
1,7	0,8162	0,8201	0,8201	0,9997	0,9998	0,9998

Note. T(0), temperature at the center of an infinite plate; and T(1), its surface temperature.

$$\frac{\partial T(x,\ y,\ \mathrm{Fo})}{\partial \mathrm{Fo}} = \Delta T(x,\ y,\ \mathrm{Fo}) + F(x,\ y,\ \mathrm{Fo}), \tag{1}$$

$$\left[\frac{\partial T(x,\ y,\ \mathrm{Fo})}{\partial \nu} + \mathrm{Bi}_m(\mathrm{Fo})\, T(x,\ y,\ \mathrm{Fo})\right]\Bigg|_{S_m} = \mathrm{Bi}_m(\mathrm{Fo})\, T_{\mathrm{a}}(\mathrm{Fo}), \tag{2}$$

$$T(x,\ y,\ 0) = \varphi(x,\ y) \tag{3}$$

for any given Bi = Bi(Fo) and $T_a = T_a(\mathrm{Fo})$, where $S = \bigcup_{m=1}^{n} S_m$ is the surface of the infinite prismatic body; $T = T^*/T_{a0}$ and $T_{a0} = 1°C$.

Employing a very simple implicit difference scheme [2] in time with a step ΔFo, we obtain a sequence of steady-state heat-conduction problems for each layer:

$$\Delta T_k - \frac{1}{\Delta \mathrm{Fo}} T_k = -\frac{1}{\Delta \mathrm{Fo}} T_{k-1} - F_k, \tag{4}$$

$$\left[\frac{\partial T_k}{\partial \nu} + \mathrm{Bi}_{mk} T_k\right]\Bigg|_{S_m} = f_{mk}, \tag{5}$$

where $T_k(x,\ y) = T(x,\ y,\ \mathrm{Fo}_k)$; $\mathrm{Bi}_{mk} = \mathrm{Bi}_m(\mathrm{Fo}_k)$.

We write the structures of the solution of problem (4), (5) for each layer in the form [3, 4]

$$T_k(x,\ y) = T_{\mathrm{a}k} + u_k = T_{\mathrm{a}k} + \sum_{i,j} C_{ij}^{(k)} \mathrm{X}_{ij}^{(k)}, \tag{6}$$

where

$$\mathrm{X}_{ij}^{(k)} = (1 + \psi_k \omega) P_i(x) P_j(y) - \omega\left(\frac{\partial \omega}{\partial x}\frac{\partial P_i}{\partial x} P_j + \frac{\partial \omega}{\partial y}\frac{\partial P_j}{\partial y} P_i\right);$$

$$\Psi_k = \left[\sum_{m=1}^{n} \mathrm{Bi}_{mk}\omega_m^{-1}\right]\left[\sum_{m=1}^{n} \omega_m^{-1}\right]^{-1};$$

$$\omega_m\Big|_{S_m} = 0;\quad \omega\Big|_S = 0;\quad \omega_m > 0;\quad (x,\ y) \in \Omega;\quad \frac{\partial \omega}{\partial \nu}\Big|_S = -1;$$

ν is the outward normal to surface S.

TABLE 2. Approximate and Exact Solutions

Fo	T (0; 0)		T (1; 0)		Fo	T (0; 0)		T (1; 0)	
	T_{exact}	$T_{approx.}$	T_{exact}	$T_{approx.}$		T_{exact}	$T_{approx.}$	T_{exact}	$T_{approx.}$
0,001	3,265	3,311	2,505	2,526	0,061	4,692	4,778	2,925	2,957
0,011	3,429	3,478	2,559	2,581	0,071	5,074	5,171	3,022	3,056
0,021	3,616	3,670	2,618	2,642	0,081	5,517	5,629	3,130	3,167
0,031	3,830	3,891	2,684	2,709	0,091	6,034	6,162	3,249	3,289
0,041	4,078	4,145	2,756	2,783	0,096	6,324	6,462	3,313	3,355
0,051	4,362	4,438	2,836	2,865					

We obtain the following problems for the functions u_k:

$$\Delta u_k - \frac{1}{\Delta \mathrm{Fo}} u_k = \frac{1}{\Delta \mathrm{Fo}} T_{ak} - \frac{1}{\Delta \mathrm{Fo}} T_{k-1} - F_k = -F_k^*, \tag{7}$$

$$\left[\frac{\partial u_k}{\partial \nu} + \mathrm{Bi}_{mk} u_k\right]\Big|_{S_m} = 0. \tag{8}$$

We find the coefficients $C_{ij}^{(k)}$ from the condition for the minimum of the functional

$$I(u_k) = \iint_\Omega \left[(\mathrm{grad}\, u_k)^2 + \frac{1}{\Delta \mathrm{Fo}} u_k^2 - 2F_k^* u_k\right] d\Omega + \sum_{m=1}^{n} \int_{S_m} \mathrm{Bi}_{mk} u_k^2 dS_m.$$

For an infinite plate $-\infty < z < \infty;\ -\infty < y < \infty;\ -1 \le x \le 1$ we obtain from (6)

$$X_i^{(k)} = \Phi_i - \omega \frac{d\Phi_i}{dx}\frac{d\omega}{dx} + \omega\, \mathrm{Bi}\,(\mathrm{Fo}_k)\,\Phi_i;$$

$$\Phi_i = x^{2i-2},\ i = 1, \ldots, n;\ \Phi_{0k} = T_a(\mathrm{Fo}_k);\ \omega = \frac{1}{2}(1 - x^2).$$

The values of the temperature for Bi = 0.5 exp (Fo), $T_a = 1 + 0.075\mathrm{Fo}$, and $\varphi = 0.15$ calculated on a BÉSM-6 computer by the proposed method (two coordinate functions), by the method of elementary balances [5], and by using data in [6] are listed in columns one, two, and three respectively in Table 1.

To test the efficiency of the proposed approach to the solution for small values of Fo (initial period) we consider problem (1)-(3) for a rectangular prism $-1 \le x \le 1;\ -1 \le y \le 1;\ -\infty < z < \infty$ for $\varphi = 1 + (1 + f_1)(1 + f_2)$:

$$\mathrm{Bi} = \exp(10\,\mathrm{Fo});\ T_a = 1 + 0.075\,\mathrm{Fo};\ F = \exp(10\,\mathrm{Fo})(1 + 10 f_2) \times$$
$$\times [1 + f_1 \exp(10\,\mathrm{Fo})] + \exp(10\,\mathrm{Fo})(1 + 10 f_1)[1 + f_2 \exp(10\,\mathrm{Fo})] + 0.075.$$

It is easy to verify that in this case the exact solution of problem (1)-(3) is

$$T(x, y, \mathrm{Fo}) = T_a(\mathrm{Fo}) + [1 + f_1 \exp(10\,\mathrm{Fo})][1 + f_2 \exp(10\,\mathrm{Fo})], \tag{9}$$

where $f_1 = 0.5(1 - x^2)$; $f_2 = 0.5(1 - y^2)$.

In this case in the structures of solution (6)

$$T_{ak} = T_a(\mathrm{Fo}_k),\ X_{ij}^{(k)} = \{1 + \omega [f_2 \exp(10\,\mathrm{Fo}_k) + f_1 \exp(10\,\mathrm{Fo}_k)]\ (f_1 + f_2)^{-1}\} P_i P_j - \omega\left(\frac{\partial \omega}{\partial x}\frac{\partial P_i}{\partial x} P_j + \frac{\partial \omega}{\partial y}\frac{\partial P_j}{\partial y} P_i\right),$$

where $P_i(x)$ and $P_j(y)$ are Chebyshev polynomials, and $\omega = f_1 + f_2 - \sqrt{f_1^2 + f_2^2}$.

Table 2 lists the values of T_{approx} calculated from Eqs. (6) (fifteen coordinate functions) and the exact values T_{exact} (9) for $0.001 \le \mathrm{Fo} < 0.1$.

When employing the proposed method it is sufficient to construct the function ω for plates and infinite prismatic bodies of complex cross section by using the recommendations in [7].

A comparison of a test example for a square prism with the exact solution (9) is of independent interest also for numerical methods: Bi(Fo) in the initial period $(0 < \mathrm{Fo} < 0.1)$.

NOTATION

T, temperature of plate or infinite prismatic body; Bi, Biot number; T_a, ambient temperature; $F = W/\lambda$; W, specific strength of energy sources; λ, thermal conductivity.

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MAIN TYPES OF CONJUGATE PROBLEMS IN HEAT AND MASS EXCHANGE*

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The development of optimal technological processes and apparatuses for heat and mass exchange is placing more and more severe requirements on the suitability of mathematical models and the accuracy of their mechanization. Of great interest in this connection is the solution of problems in heat and mass exchange in the conjugate formulation, which makes it possible to take account more fully of the interrelation between the transfer processes taking place in the bodies which are in contact. In this case the description of the transfer processes at the interface between the phases makes use of boundary conditions of the fourth kind, which are differential equations that arise out of the laws of conservation of the relevant entities being transferred – energy, mass, momentum, etc.

Although boundary conditions of the fourth kind have been used for a fairly long time [1], the solution of problems in heat and mass exchange in the conjugate formulation was long restricted by the inadequate level of development of the analytic and numerical methods of solution. The number of published works devoted to the solution of conjugate problems (CP) in heat and mass exchange began to increase rapidly after the appearance of the works of Lykov and Perel'man [2, 3], which were the first to formulate an external CP in heat exchange and show the desirability of such a formulation. The rapid spread of investigations related to conjugate heat- and mass-exchange problems was greatly facilitated by the development of numerical methods of solution designed for use with computers.

The place occupied by CP in the theory of heat and mass exchange is similar in many ways to the position occupied by boundary-layer theory, ideal-liquid theory, mechanics of viscous continuous media, or

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